CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the modulus and amplitude of $\frac{(3-\sqrt{2}i)^2}{1+2i}$. **(06 Marks)**

b. Find the cube root of $(1-i)$. **(05 Marks)**

c. Prove that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(n\frac{\pi}{2}-n\theta\right)+i\sin\left(n\frac{\pi}{2}-n\theta\right)$. **(05 Marks)**

OR

2 a. For any three vector a, b, c show that
$\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ **(06 Marks)**

b. Find the value of λ so that the vectors $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=\hat{j}+\lambda\hat{k}$ are coplanar. **(05 Marks)**

c. Find the angle between the vectors $\vec{a}=5\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$ **(05 Marks)**

Module-2

3 a. Find the n^{th} derivative of $\cos x\cos 2x\cos 3x$. **(06 Marks)**

b. If $y = a\cos(\log x)+b\sin(\log x)$, prove that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n = 0$. **(05 Marks)**

c. Find the angle between the radius vector and tangents for the curve $r^2\cos 2\theta = a^2$ **(05 Marks)**

OR

4 a. If $u = e^{ax+by}+(ax-by)$ prove that $b\frac{\partial u}{\partial x}+a\frac{\partial u}{\partial y} = 2abu$. **(06 Marks)**

b. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \tan u$. **(05 Marks)**

c. If $x = u(1-v)$, $y = uv$. Find $\frac{\partial(x,y)}{\partial(u,v)}$. **(05 Marks)**

Module-3

5 a. Obtain the reduction formula for $\int_0^{\frac{\pi}{2}}\cos^n x\,dx$ (n>0). **(06 Marks)**

b. Evaluate $\int_0^1 x^6\sqrt{1-x^2}\,dx$. **(05 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^y xyz\,dx\,dy\,dz$. **(05 Marks)**

OR

6 a. Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$, $n > 0$. **(06 Marks)**

b. Evaluate $\int_0^a x^2(a^2 - x^2)^{\frac{3}{2}} dx$. **(05 Marks)**

c. Evaluate $\int_0^1 \int_0^{\sqrt{x}} xy dy dx$. **(05 Marks)**

Module-4

7 a. A particle moves along a curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ where t is the time. Determine the component of velocity and acceleration vector at t = 0 in the direction of $\hat{i} + \hat{j} + \hat{k}$. **(08 Marks)**

b. Find the value of the constant a, b, such that $\vec{F} = (axy + z^3)\,\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational. **(08 Marks)**

OR

8 a. If $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$ show that $\vec{F} \cdot \text{curl}\,\vec{F} = 0$. **(06 Marks)**

b. If $\phi(x, y, z) = x^3 + y^3 + z^3 - 3xyz$ find $\nabla\phi$ at $(1, -1, 2)$. **(05 Marks)**

c. Find the directional derivative $\phi(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **(05 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y}{x - \sqrt{xy}}$. **(06 Marks)**

b. Solve $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$ **(05 Marks)**

c. $\frac{dy}{dx} - \frac{2y}{x} = x + x^2$. **(05 Marks)**

OR

10 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$. **(06 Marks)**

b. Solve $(y^3 - 3x^2y)dx - (x^3 - 3xyz)dy = 0$ **(05 Marks)**

c. Solve $(1 + y^2)dx + (x - \tan^{-1} y)dy = 0$ **(05 Marks)**

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